The Different Effect of Consumer Learning on Incentives to Differentiate in Cournot and Bertrand Competition*

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Abstract

We combine two extensions of the differentiated duopoly model of Dixit (1979), namely Caminal and Vives (1996) and Brander and Spencer (2015a,b), to analyze the effect of consumer learning on firms' incentives to differentiate their products in models of Cournot and Bertrand competition.

Products are of different quality, consumers buy sequentially and are imperfectly informed about the quality of the goods. Before simultaneously competing in quantities, firms simultaneously choose their investment into differentiation. Late consumers can observe earlier consumers' decisions and extract information about the quality of the goods. This influences the firms' incentives to differentiate.

If firms compete in quantities, they are more likely to invest in differentiation with consumer learning than without. This is in line with implications of the recommendation effect introduced in Conze and Kramm (2016) in a model of spatial differentiation. We also examine the case in which firms compete in prices. Here, the effect of consumer learning is reversed, so that differentiation is less likely with consumer learning. Thus, we find an information-based difference between Cournot and Bertrand competition: in the Bertrand setting consumer learning increases the competition, i.e. products are more likely to be substitutes, and it weakens it in the Cournot model.

Keywords: Principle of Minimum Differentiation, Consumer Learning, Bayesian Observational Learning

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1 Introduction

Most of the literature dealing with firms' incentives to differentiate characterizes two different and opposing effects. The competition effect induces firms to differentiate their products from each other, since they then obtain local monopoly power and are able to charge higher prices. On the other hand, differentiating decreases the market share and the amount of goods that the firm in question is able to sell. This is the so-called market-size effect (see e.g. Belleflamme and Peitz, 2010, Chapter 5.2).

In the related research project Conze and Kramm (2016) we use a spatial model of product differentiation à la Hotelling (1929) to establish a new effect that may incentivize firms to differentiate. The effect arises because of the possibility of consumer learning, and is called the recommendation effect. Its intuition is as follows. In the model, two firms A and B compete by choosing their locations on the unit interval, representing the choice of the goods' characteristics. Two consumers sequentially choose between the two goods that are of different quality. Consumers are heterogeneous with respect to their preference towards the goods and to their information about the quality differential. The late consumer (laggard) observes the purchase decision of her predecessor (early adopter), which may contain valuable information. Neither the information, nor the preference of the early adopter is observed by the laggard. The laggard then uses Bayes' Rule to update her belief on the good's qualities. In this setup, it is the case that a purchase of a niche (i.e. a differentiated) product in the first period is more likely based on its high quality than on a good match of consumer taste and product characteristic. A firm can influence and exploit consumer learning using its location choice (mainstream vs. niche) which yields incentives to offer a differentiated (niche) product.

In order to make the above mentioned model tractable, it abstracts from endogenous prices by assuming that they are regulated to the same value for both firms. Additionally, there are only two consumers and two states of the world, either good A's quality exceeds the one of good B by a fixed amount, or vice versa. Although there are situations plausibly described with these assumptions, the goal of the research at hand is to show that similar effects as described above also arise in a model where these assumptions are relaxed.[1]

[1] More explicitly, the model at hand differs from Conze and Kramm (2016) in that it entails endogenous prices, a simultaneous choice of product differentiation, a continuously distributed quality differential, a continuous information structure and the fact that firms may reset one of their choice variables (quantities in

Underlying the model in the paper at hand is the standard model of differentiated duopoly introduced by Dixit (1979). Dixit's model was extended in various ways. In particular, Caminal and Vives (1996) formulate a model of Bertrand competition with two firms who compete for a continuum of consumers in each of two periods by setting prices. The goods of the two firms are horizontally and vertically differentiated, but firms can not control either dimension. Firms also do not know the quality of their good when setting their prices. Consumers receive signals about the goods' qualities, and the consumers in the second period observe past market shares but not past prices. The authors show that such a situation leads to lower prices in the first period - compared to a situation without consumer learning - as each firm has an incentive to decrease its price in order to obtain a higher market share. This is because a high market shares serves as a signal of high quality to consumers in the second period. The authors state that "[...] an increase in the degree of product substitutability [...] increases the effectiveness of the manipulation by firms" (Caminal and Vives, 1996, p. 228). As the model does not allow for endogenous levels of differentiation or substitutability, Caminal and Vives do not elaborate on this insight any further.

This is where the model of Brander and Spencer (2015a,b), also based on Dixit's model, comes in. In their papers, the authors analyze the competition between two firms in the common differentiated duopoly setup without vertical differentiation. The goal of their article is to compare the firms' incentives to differentiate in Cournot and Bertrand competition setups. To endogenize the levels of differentiation, the authors assume that firms can make costly investments in order to increase the differentiation between the products. It is shown that differentiation is more likely to occur in Bertrand than in Cournot setups.

We combine the approaches of Caminal and Vives (1996) and Brander and Spencer (2015a) to explore how the possibility of consumer learnings influences firms incentives to differentiate their products. Our model thus makes the following changes to the differentiated duopoly setup of Dixit (1979): First, we allow the firms' products to be of different, random and a priori unknown quality, which introduces information asymmetries. The model consists of three stages. Before firms compete in quantities, they decide on their investment into differentiation. This stage is followed by two stages in which firms set quantities and

Cournot, prices in Bertrand competition), which would be equivalent to allowing firms to relocate in Conze and Kramm (2016).

consumers buy the goods, based on imperfect signals about the goods qualities. In the last stage, consumers additionally observe past prices (but not market shares).

Additionally, we analyze an analogous Bertrand model with consumer learning and compare the implications of Bayesian updating among consumers between the two models of price and quantity competition. We start with analyzing Cournot competition as in this model the endogenous variable of interest, namely the product differentiation, appears more intuitively in the utility function of the consumers.

2 Model Setup: Quantity Competition

In two periods, $t = 1, 2$, two firms, $j = A, B$, compete by producing quantities x_t^j. In each period there is a continuum of consumers with mass one uniformly distributed on $[0, 1]$ and indexed by i. Consumers have access to distinct information. The utility of the consumers when consuming any real-valued amount $x = (x^A, x^B, x^0) \in \mathbb{R}^3$ is given by

$$U(x^A, x^B, x^0) = (\alpha + q)x^A + (\alpha - q)x^B - 0.5\left[(x^A)^2 + 2\gamma x^A x^B + (x^B)^2\right] + x^0$$

with $\alpha > 0$. The idea is that all consumers agree on the utility of the goods and thus have the same utility function, which can be derived from quasilinear preferences.[2] Quantity x^0 captures the consumption of a composite good containing all other goods different from x^A and x^B, and its price is normalized to $p^0 = 1$. The feasible range of the measure of product differentiation or substitutability is $\gamma \in (0, 1]$. The higher γ, the higher is the substitutability between the products. Goods are perfect substitutes if $\gamma = 1$ and they would be independent if $\gamma = 0$.

The (relative) quality of the goods is measured by the random variable q, which is normally distributed with mean zero and variance $1/\tau_q$.[3] Variable q is unknown to firms and consumers, but each consumer receives a signal $s_t^i = q + \epsilon_t^i$ about it, where ϵ again is an independent and

[2] In addition to heterogeneous information levels, we could introduce heterogeneity in the utility by choosing an individual parameter α_i with an appropriate distribution such that the results continue to hold. One could interpret each representative agent with a certain information level as representing a group of consumers with that same information level.

[3] As will become clear later on, when dealing with the rational updating of the consumers, it is easier to work with precisions τ than with variances.

normally distributed random variable with zero mean and variance $1/\tau_\epsilon$.[4] Both variances, $1/\tau_q$ and $1/\tau_\epsilon$, are known to the players in the model. We assume $\int_0^1 s_t^i di \xrightarrow{a.s.} q,\ t \in \{1,2\}$, analogously to a version of the Strong Law of Large Numbers.[5] The consumers in period two also observe past prices, but not sold quantities or previous signals. This is the case for instance at online platforms on which consumers can observe the history of past prices and current quantities, but not quantities sold in previous periods. Let I_t^i denote all available information to consumer i in period t.

Rational consumers maximize their expected utility subject to the budget constraint $m = x^0 + p^A \cdot x^A + p^B \cdot x^B$, which yields the individual demand[6]

$$\begin{aligned}
x_t^{A,i} &= \frac{\alpha}{1+\gamma} + \frac{s_t^i}{1-\gamma} - \frac{p^A}{1-\gamma^2} + \frac{\gamma p^B}{1-\gamma^2},\\
x_t^{B,i} &= \frac{\alpha}{1+\gamma} - \frac{s_t^i}{1-\gamma} - \frac{p^B}{1-\gamma^2} + \frac{\gamma p^A}{1-\gamma^2},\\
x_t^{0,i} &= m - p_A \cdot x^{A,i} - p_A \cdot x^{B,i}.
\end{aligned}$$

Aggregating the rational consumers' demands via $x_t^j = \int_0^1 x_t^{j,i} di$ and inverting yields the total inverse demand for the product of firm $j \in \{A,B\}$ generated by rational consumers. In addition to the rational consumers, there are also consumers who ignore prices and whose

[4] A strand of recent literature deals with the implications of unknown quality to both buyers and sellers, see e.g. Szentes and Roesler (2016).

[5] This assumption needs to be made due to a related issue pointed out by Judd (1985).

[6] As in many economic models, the application of normally distributed random variables has economically implausible consequences: depending on the received signal, individual demand might become negative or tend to infinity for a fixed set of prices. Analogous consequences can be found for individual inverse demand for a fixed set of quantities in Bertrand competition. To rule out such cases one could alternatively assume that q is distributed according to a truncated normal distribution on an interval $[-Y, Y]$ with $0 < Y < \alpha/(1-\gamma^2)$. One would then only need to incorporate the changed variance of q, the rest of the updating remains unchanged. Note, that a truncated prior implies that the posterior is truncated at the same values independent of the distribution of the signal. Another possibility to decrease the probability of such 'unwanted' events would be to increase the precision of the random variables. Our results hold when we approach the model without asymmetric information and consumer learning, i.e. when $\tau_\epsilon \to \infty$.

If $\gamma = 1$, the stated individual demands become infinitely large or small. Furthermore, in the analogous model of Bertrand competition, the consumers' first order conditions are not invertible, such that there are problems with the microfoundation of the aggregate model described in Section 4. However, when starting directly with the standard aggregated form of the oligopoly models as represented in equations (1) and (7), such issues are avoided.

utility for both products is the same irrespectively of the realized quality, in each period. Those consumers purchase from both firms randomly (see also Caminal and Vives, 1996). Their impact on the inverse demand at firm j in period t is the given by the random variable u_t^j. These random variables are i.i.d. draws from $\mathcal{N}(0, 1/\tau_u)$.[7]

Let $\eta_t := \int_0^1 E(q|I_t^i)di$ denote the aggregate belief on quality of all rational consumers in period t. Combining the demand of rational consumers and random shoppers leads to the following aggregated inverse demand functions

$$\begin{aligned} p_t^A &= \alpha + \eta_t - x_t^A - \gamma x_t^B + u_t^A, \\ p_t^B &= \alpha - \eta_t - x_t^B - \gamma x_t^A + u_t^B. \end{aligned} \tag{1}$$

The product substitutability γ is endogenous and chosen by the firms via their investment decision. An example is the investment of Coca Cola and Pepsi into advertisement, in order to emphasize the differences between the two products, although their taste is indistinguishable for consumers (see Brander and Spencer, 2015a,b). At the beginning of the game, in period $t = 0$, firms can make monetary investments k^A and k^B in order to increase the differentiation between their products, according to the following functional form

$$\gamma = e^{-\lambda(k^A + k^B)}.$$

If both firms make zero investments, then $\gamma = 1$ and the goods become perfect substitutes. Larger investments of any firm decrease γ, which approaches zero if the investments approach infinity. Of course there are cases in which firms invest into making their products complementary to each other, which however is not modeled in our setup. As $\gamma \in (0, 1]$, the goods are between the extremes of independence or perfect substitutability.[8] Cases where the goods are or can become complements are left out here, as our focus is on situations like the Coke-Vs-Pepsi story mentioned above. The parameter $\lambda \in \mathbb{R}_+$ measures the technology or effectiveness of how the firms' investments translate into increased differentiation.

[7] The fact that irrational consumers may have a negative impact on (inverse) demand might be interpreted as some part of the rational consumers refraining from a purchase of the good, although rational utility optimization implied the opposite.

[8] Our model is a special case of a specification using the utility function $U(x^A, x^B, x^0) = (\alpha + q)x^A + (\alpha - q)x^B - 0.5\left[\beta(x^A)^2 + 2\gamma x^A x^B + \beta(x^B)^2\right] + x^0$ with $\beta = 1$. In the more general model goods may also be perfect complements, i.e. $\gamma = -1$.

The higher λ, the smaller are the necessary investments to increase the differentiation by a certain amount.

Firms maximize their expected total profits and second period profits are discounted with the factor $\delta \in (0, 1)$. Note that firms do not know the quality (differential) when making their differentiation investments and choosing their quantities, which is plausible for instance in the case of experience goods. Thus, their information set in the first two periods is given by $I_0 = I_1 = \{\emptyset\}$, and by $I_2 = \{x_1, p_1\}$ in the last period.

As it is common for Cournot models, the process of price formation can be modeled via an auctioneer. She knows quality q, which allows her to calculate the consumers' aggregate belief on quality η_t (see below), and she also knows the realization of u_t^A and u_t^B for $t = 1, 2$. In each period, the firms inform the auctioneer about the quantity produced and the auctioneer calculates p_t^A and p_t^B. These prices are announced to consumers and each of them purchases his optimal quantity of each of the two goods. As announced prices contain information on the quality, we assume that consumers' beliefs are not affected by announced prices. This assumption can be justified by the fact that consumers do not understand the (informationally complex) process of price formation implemented by the auctioneer.

The alternative justification of Cournot models pioneered by Kreps and Scheinkman (1983), a model with a stage where both firms choose capacities before competing in prices, and which under certain assumptions leads to 'standard' Cournot outcomes, can not be applied here. This is the case as introducing demand uncertainty in a capacity-then-price-competition model leads to non-existence of a pure strategy equilibrium, in particular also to the absence of the equilibrium with Cournot quantities (see for example Hviid, 1991 and Behrens and Lijesen, 2012). As the case of homogeneous goods is nested in our model, the same applies here.

For convenience we let firm specific variables without superscript denote the vector of the two variables of both firms and by Δy we describe the difference of variable y^A and y^B, so for example $x_1 = (x_1^A, x_1^B)$ and $\Delta u_1 = u_1^A - u_1^B$. The following graphic depicts the timing of the game:

$t = 0$

- Quality q realizes.
- Firms make their differentiation investments k^A and k^B simultaneously knowing $I_0 = \emptyset$.

$t = 1$

- Firms set quantities x_1^A, x_1^B knowing $I_1 = \emptyset$.
- Consumer i receives signal s_1^i
- (• Auctioneer announces prices p_1^A and p_1^B knowing η_1 and u_1)
- Consumer i purchases optimal quantity of goods knowing $I_1^i = \{s_1^i, x_1\}$.

$t = 2$

- Firms set quantities x_2^A, x_2^B knowing $I_2 = \{p_1, x_1\}$.
- Consumer i receives signal s_2^i
- (• Auctioneer announces prices p_2^A and p_2^B knowing η_2 and u_2)
- Consumer i purchases optimal quantity of goods knowing $I_2^i = \{s_2^i, \Delta p_1, x_2\}$.

Figure 1: Timing of the Game

We employ the solution concept of Perfect Bayesian Equilibrium. To avoid complications off the equilibrium path, it is assumed that consumers' beliefs are constant with respect to observed current-period quantities, i.e. $\partial \eta_t^i / \partial x_t = 0$.

3 Solving the Model with Quantity Competition

We first replicate and adapt the results for Bertrand competition of Caminal and Vives (1996) for our Cournot framework. Then we analyze the effect of consumer learning on product differentiation in this framework.

3.1 Consumers

In order to characterize the optimal behavior of consumers, we only need to calculate how they use the available information to update their beliefs about the goods' qualities and can then use the aggregated inverse demand calculated in equation (1). We exploit the properties of the normal distribution, in particular the fact that the updating rules for both, mean and variance, are linear (see e.g. Section 2.2.2 of Chamley, 2004). Details on the calculations can be found in Appendix A.

First Period Early adopter i's belief on the quality is given by $\eta_1^i := E[q|I_1^i] = m_1 s_1^i$ where $m_1 := \frac{\tau_\epsilon}{\tau_\epsilon + \tau_q}$ weighs up the precisions of the distributions of the signal the consumers receive against the precision of the quality. The aggregate expectation in the first period can be calculated as

$$\eta_1 = \int_0^1 \eta_1^i di = m_1 q.$$

As $\mathrm{Var}(\eta_1) < \mathrm{Var}(q)$, some uncertainty is resolved in the aggregate and in aggregate the consumers' belief is closer to the true value of q than the unconditional expectation $E[q]$. In equilibrium beliefs are correct, such that the consumers' first period equilibrium belief is $\eta_1^* = m_1 q$.

The utility maximization of the rational consumers and the behavior of the random shoppers results in the following inverse demand

$$p_1^A = \alpha + m_1 q - x_1^A - \gamma x_1^B + u_1^A,$$
$$p_1^B = \alpha - m_1 q - x_1^B - \gamma x_1^A + u_1^B.$$

Second Period In addition to the signal and the quantities x_2, the laggard i's information set now also contains the observed price difference in period 1, i.e. her information set is $I_2^i = \{s_2^i, \Delta p_1, x_2\}$. The price differential Δp_1 contains information about q:

$$\Delta p_1 := p_1^A - p_1^B = 2m_1 q - (1-\gamma)\Delta x_1 + \Delta u$$
$$\Leftrightarrow q = [\Delta p_1 + (1-\gamma)\Delta x_1 - \Delta u_1]/2m_1.$$

As Δx_1 is not observed by consumers in the second period and $E[\Delta u_1] = 0$, a laggard's best estimate of the quality is $q^e = [\Delta p_1 + (1-\gamma)\Delta x_1^e]/2m_1$, where Δx_1^e is the conjectured difference in quantities. Because actual first period quantities are not observed by consumers in the second period, they have to make conjectures about them, that is, they need to interpret past prices as signals of the chosen quantities, which is formalized by Δx_1^e. Thus, q^e is obtained by solving the observed price difference Δp_1 for q and replacing the unknown variables from the perspective of the consumer by their expected value (Δu_1) and the conjecture about the played strategy (Δx_1). Inserting the realized price difference Δp_1, it equals

$$q^e = q + [\Delta u_1 - (1-\gamma)(\Delta x_1 - \Delta x_1^e)]/2m_1. \tag{2}$$

This expression contains the two random variables q and Δu_1, and the second summand captures the error the consumers make in conjecturing past market shares.

The laggard now combines his observation extracted from the price difference with his signal using Bayesian updating, so that her belief is given by $\eta_2^i := E[q|I_2^i] = m_2 s_2^i + n_2 q^e$, with $m_2 = \tau_\epsilon/\tau_2^i$, $n_2 = 2\tau_u m_1^2/\tau_2^i$ and $\tau_2^i = \tau_\epsilon + \tau_q + 2\tau_u m_1^2$. The aggregate belief then is given by

$$\eta_2 = \int_0^1 \eta_2^i di = m_2 q + n_2 q^e.$$

As in equilibrium beliefs are correct we obtain the equilibrium belief $\eta_2^* = m_2 q + n_2 \tilde{q}$ with $\tilde{q} = q + \Delta u_1/2m_1$. Note that $\tilde{q}$ equals q^e with correctly conjectured first period quantities, i.e. with $\Delta x_1^e = \Delta x_1$.

Clearly,

$$\frac{\partial \eta_2}{\partial x_1^A} = -\frac{\partial \eta_2}{\partial x_1^B} = n_2 \cdot \frac{\partial q^e}{\partial x_1^A} = -\frac{(1-\gamma)n_2}{2m_1}. \tag{3}$$

The derivative shows that the effect of a change in the first period quantity on the consumer belief in the second period is higher, the smaller the substitutability between products, γ. Phrased differently, the more differentiated the goods are, the higher is the impact of a firm's change of its choice variable in the first period on the laggards' belief. The decreased price p_1^A induced by a higher quantity x_1^A decreases the belief that product A is of superior quality because first period quantities are not observed by laggards, so that consumers can not be certain whether the price decrease was due to a low quality product, or due to a high volume of sales. This reasoning is analogous to the recommendation effect as introduced in Conze and Kramm (2016).

Similarly as in period one, the utility maximization of the rational consumers and the behavior of the random shoppers results in the following inverse demand in period two:

$$p_2^A = \alpha + m_2 q + n_2 q^e - x_2^A - \gamma x_2^B + u_2^A,$$
$$p_2^B = \alpha - m_2 q + n_2 q^e - x_2^B - \gamma x_2^A + u_2^B.$$

3.2 Firms

Firm behavior is analyzed via backward induction, but we start with analyzing the firms' information processing. Details on the calculations can be found in Appendix B.

3.2.1 Bayesian Updating

In order to optimally set quantities, firms need to forecast the consumers' beliefs on quality, so they form a belief about the consumers' (aggregate) belief on quality denoted by $\theta_t := E[\eta_t|I_t]$. Both firms have identical information and so cannot manipulate each other.

In period 1, the firms do not have any information about the consumers' belief and thus $\theta_1 = E[m_1 q|I_1] = 0$.

In period 2, in contrast to the consumers, the firms can extract $\tilde{q}$ (the consumers' second period estimate of the quality extracted from the price difference in the previous period with correctly conjectured quantities) from past prices and quantities, so that $\theta_2 = E[m_2 q + n_2 q^e|I_2] = m_2 E[q|\tilde{q}] + n_2 E[q^e|\tilde{q}]$. Equilibrium beliefs are defined to be $\theta_t^* = E[\eta_t^*|I_t]$.

3.2.2 Optimal Quantities

Given the beliefs about consumers' beliefs, firms choose optimal quantities. In the second period, firms take the differentiation parameter and first period quantities and prices as given so that their optimization problem boils down to maximizing the profit $\pi_2^j = x_2^j \cdot p_2^j(x_2)$ by the choice of x_2^j. Best responses are given by $x_2^A(x_2^B) = \frac{\alpha+\theta_2-\gamma x_2^B}{2}$ and $x_2^B(x_2^A) = \frac{\alpha-\theta_2-\gamma x_2^A}{2}$. Equilibrium quantities are

$$x_2^{A*} = \frac{\alpha}{2+\gamma} + \frac{\theta_2^*}{2-\gamma},$$
$$x_2^{B*} = \frac{\alpha}{2+\gamma} - \frac{\theta_2^*}{2-\gamma}.$$

In the first period firms take into account the indirect effect their quantity choice has on the profit in period 2 via Bayesian updating among the consumers. Thus, the objective function of firm A is given by

$$\pi_1^A(x_1) = x_1^A(\alpha + \theta_1 - x_1^A - \gamma x_1^B) + \delta E[\pi_2^A|I_1], \tag{4}$$

where π_2^A is firm A's second period profit and π_1^A is the total revenue of firm A, that is the profit from periods one and two, ignoring potential investments in differentiation. Remember that $\theta_1 = 0$. Furthermore, note that $E[p_2^A|I_1] = \frac{\alpha}{2+\gamma} + \frac{\theta_2}{2-\gamma} = x_2^{A*}$, which implies that $E[\pi_2^A|I_1] = E[(x_2^{A*})^2|I_1]$. Firm A's best response is then $x_1^A(x_1^B) = \alpha/2 - x_1^B\gamma/2 + \frac{\delta x_2^{A*}}{2-\gamma} \cdot \frac{\partial\theta_2}{\partial x_1^A}$. Firm B's best response can be calculated analogously and, using equation (3), equilibrium quantities are

$$x_1^{A*} = x_1^{B*} = \frac{\alpha}{2+\gamma} \cdot \left(1 + \frac{2\delta}{4-\gamma^2} \frac{\partial \theta_2}{\partial x_1^A}\right).$$

Overall, we obtain the following result, which is analogous to the proposition for Bertrand competition in Caminal and Vives (1996).

Lemma 1. *In the equilibrium of our model, optimal quantities in period* 2 *are given by*

$$x_2^{A*} = \frac{\alpha}{2+\gamma} + \frac{\theta_2^*}{2-\gamma} \quad \textit{and} \quad x_2^{B*} = \frac{\alpha}{2+\gamma} - \frac{\theta_2^*}{2-\gamma}. \tag{5}$$

Optimal quantities in period 1 are given by

$$x_1^{A*} = x_1^{B*} = \frac{\alpha}{2+\gamma} \cdot \left(1 - \frac{\delta(1-\gamma)}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right). \tag{6}$$

The optimal second-period quantity of firm j is higher (lower) than in a standard differentiated Cournot model ($\alpha/(2+\gamma)$), if the expectation of the consumer belief is (not) in favor of firm j.[9] That is, the firm which is expected to be preferred by consumers sells a higher quantity.

As $\gamma \leq 1$, first period quantities are (weakly) lower than those without consumer learning, meaning that first period prices exceed those of standard differentiated Cournot model. This is due to consumers in period 2 only observing past prices but not quantities. A higher price thus leads them to expect the good to be of higher quality.

3.2.3 Optimal Differentiation Investments

Forecasting the resulting optimal quantities, firms choose the investment into differentiation in period zero. There exist no closed-form solutions to derive the optimal investment in differentiation, k^{j*}, and furthermore, conventional comparative static tools such as the implicit function theorem or approaches via lattice theory involve calculations, which are too computationally complex. Thus, to compare the differentiation incentives without relying on the full solution, we use the technique of Brander and Spencer (2015a), who compare the minimal effectiveness of investments in differentiation needed to induce firms to invest, that is, we derive and compare the thresholds λ so that firms investments become positive.

[9] Equilibrium quantities are positive whenever $n_2/m_1 < 4$, which is always fulfilled.

Without Consumer Learning (Benchmark) If second period consumers were to have a belief of $\eta_2 = 0$, the model in the second period is the same as the standard model of Dixit (1979), and the resulting optimal quantities would be $x_{NL}^{A*} = x_{NL}^{B*} = \alpha/(2+\gamma)$. The profit of a benchmark model with two periods without consumer learning is thus given by

$$\pi_{NL}^j = (1+\delta) \cdot E[\pi_2^j | I_1] - k^j = (1+\delta) \cdot (x_{NL}^{j*})^2 - k^j.$$

The derivative of the objective function is then

$$\begin{aligned} \partial \pi_{NL}^j / \partial k^j &= (1+\delta) \cdot \frac{d(x_{NL}^{j*})^2}{d\gamma} \cdot \frac{\partial \gamma}{\partial k^j} - 1 = (1+\delta) \cdot \frac{-2\alpha^2}{(2+\gamma)^3} \cdot (-\lambda\gamma) - 1 \\ &= (1+\delta) \cdot \frac{2\lambda\gamma\alpha^2}{(2+\gamma)^3} - 1. \end{aligned}$$

Firm j will invest in differentiation in equilibrium if

$$\partial \pi_{NL}^j / \partial k^j \big|_{\gamma=1} > 0 \Leftrightarrow \lambda > \frac{27}{2(1+\delta)\alpha^2} := \bar{\lambda}_{NL}^C.$$

The threshold without learning can also be obtained as a corollary of Proposition 4 from Brander and Spencer (2015a) by extending their model to two periods. The threshold decreases in α, as the positive effect of increased differentiation on profit is higher the higher α, so that the necessary technology ($\bar{\lambda}$) is decreasing in α. Additionally, differentiation incentives are stronger, as δ increases. This is because the gain from differentiation is higher than the costs compared to a situation with a lower δ.

With Consumer Learning With $\pi_L^j(\cdot) := \pi_1^j(\cdot) - k_j$, $j \in \{A, B\}$, and using equation (4) and the results mentioned thereafter, the derivative of the objective function is given by

$$\partial \pi_L^j(x^*) / \partial k^j = \frac{\partial \left[x_1^{j*} \{\alpha - (1+\gamma) x_1^{j*}\} \right]}{\partial k^j} + \delta \cdot \frac{d(x_2^{j*})^2}{d\gamma} \cdot \frac{\partial \gamma}{\partial k^j} - 1.$$

Firm j will invest in differentiation in equilibrium if

$$\begin{aligned} \partial \pi_L^j / \partial k^j \big|_{\gamma=1} &= \frac{\alpha^2 (n_2 \delta + 2m_1(1+\delta))\lambda}{27 m_1} - 1 > 0 \\ \Leftrightarrow \qquad \lambda &> \frac{27 m_1}{\alpha^2 (2m_1 + 2\delta m_1 + \delta n_2)} := \bar{\lambda}_L^C. \end{aligned}$$

We can easily see that $\bar{\lambda}_L^C < \bar{\lambda}_{NL}^C$, as

$$\bar{\lambda}_L^C = \bar{\lambda}_{NL}^C \frac{2\alpha^2(1+\delta)}{2\alpha^2(1+\delta) + \alpha^2 \delta n_2 / m_1} < \bar{\lambda}_{NL}^C,$$

which leads to our first main result:

Proposition 1. *In the Cournot model, in equilibrium firms offer perfect substitutes for a smaller range of parameters λ with consumer learning than without. That is, the threshold λ above which firms invest in differentiation is lower with consumer learning than without, $\bar{\lambda}_L^C < \bar{\lambda}_{NL}^C$.*

The comparative statics of threshold $\bar{\lambda}_L$ with respect to α and δ are the same as those of threshold $\bar{\lambda}_{NL}^C$, but the extent of the changes on the threshold induced by changes in α and δ now depends on the parameters introduced by consumer learning, m_1 and n_2. Furthermore, as $\frac{\partial n_2}{\partial \tau_u} > 0$ and $\frac{\partial \bar{\lambda}_L}{\partial n_2} < 0$, the critical value $\bar{\lambda}_L$ decreases in τ_u, $\frac{\partial \bar{\lambda}_L}{\partial \tau_u} < 0$. Intuitively, the more noise caused by the random shoppers is contained in the observed statistic about the quality, the smaller are the incentives to differentiate.

4 Solving the Model with Price Competition

As shown by Singh and Vives (1984), with a linear quadratic utility function, there is a close relationship between Cournot and Bertrand competition. In their words:

> Cournot (Bertrand) competition with substitutes is the dual of Bertrand (Cournot) competition with complements. This means that they share similar strategic properties. For example, with linear demand, reaction functions slope downwards (upwards) in both cases. It is a matter of interchanging prices and quantities. (Singh and Vives, 1984, p. 547)

Indeed, using the following utility function from Caminal and Vives (1996) which slightly differs from the one of the previous sections,

$$U^B(x^A, x^B, x^0) = (\alpha + (1-\gamma)q)x^A + (\alpha - (1-\gamma)q)x^B - 0.5\left[(x^A)^2 + 2\gamma x^A x^B + (x^B)^2\right] + x^0,$$

and including random shoppers, similar to the model before, we obtain the following set of demand functions

$$\begin{aligned} x_t^A &= a + \eta_t - bp_t^A + cp_t^B + u_t^A, \\ x_t^B &= a - \eta_t - bp_t^B + cp_t^A + u_t^B, \end{aligned} \tag{7}$$

where $a = \alpha/(1+\gamma)$, $b = 1/(1-\gamma^2)$ and $c = \gamma/(1-\gamma^2)$. Variable η_t again is the aggregate belief about the quality of consumers in period t. Besides the slightly different utility function and induced demands, all variables remain as in the previous section.

Comparing the above system of direct demands in (7) to the inverse demand system from equation (1), we can obtain one from the other by simply exchanging quantities and prices and replacing a by α, b by $\beta = 1$ and c by $-\gamma$.

Using equation (3), this implies that the recommendation effect in the Bertrand model is formalized by

$$\frac{\partial \eta_2}{\partial p_1^A} = -\frac{\partial \eta_2}{\partial p_1^B} = n_2 \cdot \frac{\partial q^e}{\partial p_1^A} = -\frac{n_2}{2(1-\gamma)m_1}. \tag{8}$$

This shows, that the parameter of substitution (γ), has the inverse impact on the magnitude of the recommendation effect in Bertrand competition than in Cournot competition.

Additionally, using the above shortcut, we know from Lemma 1 that the equilibrium prices in this setting are given by:

Lemma 2 (Caminal and Vives, 1996)**.** *In the equilibrium of the model with price setting, optimal prices in period* 2 *are given by*

$$p_2^{A*} = \frac{a}{2b-c} + \frac{\theta_2^*}{2b+c} \quad \textit{and} \quad p_2^{B*} = \frac{a}{2b-c} - \frac{\theta_2^*}{2b+c}. \tag{9}$$

Optimal prices in period 1 are given by

$$p_1^{A*} = p_1^{B*} = \frac{a}{2b-c} \cdot \left(1 - \frac{(b+c)b\delta}{4b^2-c^2} \cdot \frac{n_2}{m_1}\right). \tag{10}$$

Without learning, optimal prices of both firms are calculated as $p_{NL}^{A*} = p_{NL}^{B*} = \frac{a}{2b-c}$. We see that the firm with the higher perceived quality charges a higher price and in the first period both firms charge a lower price than in a model without learning.

As in the previous section, we can use the equilibrium prices to calculate equilibrium profits for a fixed γ and solve the derivative of the profit with respect to the investment k_j evaluated at $\gamma = 1$ for the λ above which firms make their investments in differentiation. Details on the calculations can be found in Appendix C. We obtain the following result:

Proposition 2. *In the Bertrand model, in equilibrium firms offer perfect substitutes for a smaller range of parameters λ without consumer learning than with consumer learning. That is, the threshold λ above which firms invest in differentiation with consumer learning ($\bar{\lambda}_L^B$) is higher than the threshold without learning ($\bar{\lambda}_{NL}^B$):*

$$\bar{\lambda}_L^B = \frac{2}{\alpha^2\left[(1+\delta) - 2\delta n_2/(3m_1)\right]} > \frac{2}{\alpha^2(1+\delta)} = \bar{\lambda}_{NL}^B.$$

5 Informational Incentives to Differentiate: Bertrand Vs. Cournot

While we should keep in mind, that the parameter of substitution (γ) is incorporated in a different manner in the microfoundation of the Bertrand and the Cournot model,[10] it is nevertheless worthwhile to compare the influence of consumer learning on the incentives to differentiate of the two models. Comparing our findings in the different models yields our final main result:

Proposition 3. *The effect of consumer learning on the firms' incentives to differentiate their products is different in the Cournot model and in the Bertrand model. In contrast to quantity competition, consumer learning in a model with price setting decreases the firms' incentives to differentiate:*

$$\bar{\lambda}_L^B - \bar{\lambda}_{NL}^B > 0 > \bar{\lambda}_L^C - \bar{\lambda}_{NL}^C.$$

Consumer learning thus tends to increase the competition in the Bertrand setting and it weakens it in the Cournot model.

In order to understand this result in more detail, it is useful to compare the equilibrium choices from the models with learning to those without. From the perspective of period zero, where firms choose their differentiation investments, and given the equilibrium strategies for periods one and two, the expected optimal quantities in period 2 are the same in the models with and without learning as $E(\theta_2) = 0$. Thus, the second period affects the differentiation incentives only through its influence on the optimal first period choices of the firms. In the Cournot game, equilibrium quantities in the first period are given by

$$x_1^{j*} = x_{NL}^{j*} \cdot \left(1 - \frac{\delta(1-\gamma)}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right) = \frac{\alpha}{2+\gamma} \cdot \left(1 - \frac{\delta(1-\gamma)}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right)$$

and equilibrium prices in the Bertrand model are

$$p_1^{j*} = p_{NL}^{j*} \cdot \left(1 - \frac{(b+c)b\delta}{4b^2-c^2} \cdot \frac{n_2}{m_1}\right) = \frac{a}{2b-c} \cdot \left(1 - \frac{(b+c)b\delta}{4b^2-c^2} \cdot \frac{n_2}{m_1}\right) = \frac{\alpha(1-\gamma)}{2-\gamma} \cdot \left(1 - \frac{(1+\gamma)\delta}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right)$$

for j $\in \{A, B\}$. In both models, the first factor, gives the optimal choices in a model without consumer learning. The second factor in both cases is (weakly) smaller than one, so that

[10] The different utility functions in the two models are employed, as they allow to compare the impact of γ on the aggregated (inverse) demand systems in equations (1) and (7) more easily.

quantities in a Cournot style competition and prices in the Bertrand variant of our model are (weakly) decreased by the introduction of consumer learning. Only if $\gamma = 1$, the optimal choices in the models with learning and those without coincide.

Starting with the Cournot model and comparing the marginal profit of investing in differentiation (increasing k^A or k^B) at a situation where $\gamma = 1$ ($k^A = k^B = 0$), the previous calculations showed that the marginal profit is higher with learning than without, leading to the lower threshold value in the model with learning compared to the model without.

The mechanics behind this difference are the following: as in the Cournot model without learning, and in any similar model, it is the case that the two firms could increase their first period profit by reducing their quantities. In the model without learning, decreasing one's quantity below $x_{NL}^{A*} = x_{NL}^{B*}$ is not individually rational. If $\gamma < 1$, consumer learning however introduces an incentive to decrease first period quantities below the level of a model without learning due to the recommendation effect, meaning that at $\gamma = 1$, consumer learning generates an additional incentive to invest in differentiation, as this enhances the impact of the recommendation effect.

The situation in the Bertrand setup is different in that prices are already too low in the model without learning if the goal is to maximize the joint first period profit of the firms. Firms could therefore increase their profits if they were to jointly raise their prices. With $\gamma = 1$ prices in the model with and without learning coincide and equal zero. Decreasing γ, that is increasing the differentiation, increases the optimal first period price, but the increase is smaller with consumer learning. The marginal profit of increasing $k^A = k^B = 0$, is thus higher in the model without learning than it is in the model with learning, explaining the ordering of the thresholds in this setup.

Finally, we can elaborate on the result of Brander and Spencer (2015a,b) who showed that firms are more likely to invest in differentiation in Bertrand than in Cournot competition. As our benchmark models without learning are two-period extensions of their models, we obtain the same result if we compare the models without learning, that is $\bar{\lambda}_{NL}^{C} > \bar{\lambda}_{NL}^{B}$. Consumer learning has been shown to decrease the threshold in the Cournot model and increase it in the Bertrand setting, but even then, the ranking of the two models is maintained, i.e. we also have $\bar{\lambda}_{L}^{C} > \bar{\lambda}_{L}^{B}$.

6 Conclusion

Differentiating one's product from those of a competitor results in a weaker competition and thus allows for higher prices and profits. By introducing consumer learning in a duopoly model with vertically differentiated goods and by endogenizing the horizontal differentiation between the products, we have shown that the incentives to differentiate are changed when consumer learning about the strength of the vertical differentiation is introduced. Furthermore, the effects created by consumer learning differ vastly between a model of quantity and a model of price competition.

In each of the two models, consumers learn from observed previous purchasing decisions. As their observations are not fully revealing all information, firms can manipulate the inference of late consumers by influencing the purchase decisions of early consumers. When setting their prices or quantities in early periods, firms take this effect of their choices on the inference of later consumers into account. Only when the two products are perfect substitutes, the presence of consumer learning does not change the firms optimal behavior compared to a model without consumer learning.

For quantity competition with differentiated products, firms optimally choose lower quantities in a model with consumer learning than in a model without. Low quantities lead to higher prices which tend to signal higher quality to later consumers. If firms compete in prices, optimal first period prices are below those of a model without consumer learning, as here higher sold quantities signal high quality to later consumers.

These 'distortions' of the optimal choices in early periods lead to different effects on the differentiation incentives of the firms induced by consumer learning. Because profits in a Cournot model can typically be increased by reducing the produced quantities, which is precisely the effect consumer learning has in our model of quantity competition, consumer learning increases the incentives to differentiate above the incentives generated by the desire to relax competition. The reverse is true in a model of Bertrand style price competition: here increasing prices would increase the profit of the firms, but consumer learning reduces the prices even further than the already strong competition in a Bertrand setup. The introduction of consumer learning thus decreases the incentives to invest in differentiation if firms compete in prices.

The presented results seem to support the notion that price competition, i.e. a game with strategic complements, leads to a stronger competition than quantity competition, that is competition with strategic substitutes, as products are more likely to be substitutes in the former oligopoly model.

Appendix A Bayesian Updating Among Consumers

Bayes' rule in our context can be formulated as

$$f(q|o) = \frac{\phi(o|q) \cdot f(q)}{\int \phi(o|q) \cdot f(q) dq},$$

where $f(\cdot)$ is the density of q and $\phi(\cdot)$ is the density of some observation o containing information on quality q, i.e. in our case signal s_i^t or the estimate of q extracted from the price difference in the first period, q^e. Gaussian models as the one at hand, i.e. Bayesian updating over normally distributed random variables and observations, are particularly tractable, as the posterior distribution is also normal and the updating rules for mean and variance are linear: the posterior mean is the weighted average of the prior mean and that of the observation weighted with the respective precisions, while the posterior variance is that of the prior increased by that of the observation.

In our model, consumers want to best estimate q from their observations. Consumers have the prior knowledge that $q \sim N\left(\mu_q, \frac{1}{\tau_q}\right)$ and they make one or two additional observations o_r, $r \in \{1, 2\}$, with information about q. All consumers receive a signal about q and consumers in period two observe past prices. Both, the signal and the information extracted from past prices can be reformulated to observation $o_{r,t}^i$ of consumer i in period t in the following form:

$$o_{r,t}^i = q + v_{r,t}^i \quad \text{where} \quad v_{r,t}^i \sim N\left(0, \frac{1}{\tau_{v_{r,t}^i}}\right)$$

Using Bayesian updating as described above, this leads to the following distribution of q conditional on the available observations, for $t \in \{1, 2\}$

$$q|I_t^i \sim \left(\frac{\tau_q \mu_q + \sum_{r=1}^t \tau_{v_{r,t}^i} o_{r,t}^i}{\tau_q + \sum_{r=1}^t \tau_{v_{r,t}^i}}, \frac{1}{\tau_q + \sum_{r=1}^t \tau_{v_{r,t}^i}}\right)$$

Let $\eta_1^i := E[q|I_1^i]$ be the updated belief of consumer i about q in period 1 after receiving signal s_1^i, then

$$\eta_1^i \sim N\bigg((1 - m_1) \cdot 0 + m_1 \cdot s_1^i, \frac{1}{\tau_q + \tau_\varepsilon}\bigg).$$

with $m_1 := \frac{\tau_\varepsilon}{\tau_q + \tau_\varepsilon}$. Using the assumption on the average signal, the aggregate belief is given by

$$\eta_1 := \int_0^1 \eta_1^i di = \int_0^1 m_1 s_1^i di = m_1 \int_0^1 s_1^i di \to m_1 q.$$

The information a consumer i in period 2 can extract about q from the observed price difference only is given by

$$\begin{aligned} q^e &= [\Delta p_1 + (1-\gamma)\Delta x_1^e]/2m_1 \\ &= q + [\Delta u_1 - (1-\gamma)(\Delta x_1 - \Delta x_1^e)]/2m_1. \end{aligned} \tag{11}$$

This expression contains the two random variables $q \sim N(0, \frac{1}{\tau_q})$

and $\frac{\Delta u_1}{2m_1} \sim N(0, \frac{2}{4m_1^2 \tau_u})$.

When combining this with the signal, $\eta_2^i := E[q|I_2^i]$, the updated belief of consumer i about q in period 2, is normally distributed with

$$\eta_2^i \sim N\left((1 - m_2 - n_2) \cdot 0 + m_2 s_2^i + n_2 q^e, \frac{1}{\tau_2^i}\right),$$

with $\tau_2^i = \tau_\epsilon + \tau_q + 2\tau_u m_1^2$, $m_2 = \tau_\epsilon / \tau_2^i$ and $n_2 = 2\tau_u m_1^2 / \tau_2^i$ and thus the aggregate belief is given by

$$\eta_2 := \int_0^1 \eta_2^i di = \int_0^1 (m_2 s_2^i + n_2 q^e) di = m_2 \int_0^1 s_2^i di + n_2 q^e \to m_2 q + n_2 q^e,$$

again making use of the assumption on the average signal.

Appendix B Firm Behavior in the Cournot Model

Firm behavior is analyzed via backward induction.

Quantity Setting in Stage $t = 2$

Firm A's profit in stage $t = 2$ is given by

$$\pi_2^A = x_2^A \cdot p_2^A = x_2^A \cdot (\alpha + \theta_2 - x_2^A - \gamma x_2^B).$$

Best responses are obtained by the FOCs $\partial \pi_2^j / \partial x_2^j = 0$ with $j \in \{A, B\}$, which gives

$$x_2^A(x_2^B) = \frac{\alpha + \theta_2 - \gamma x_2^B}{2}, \qquad \text{and similarly} \qquad x_2^B(x_2^A) = \frac{\alpha - \theta_2 - \gamma x_2^B}{2}.$$

In equilibrium best responses intersect, so we obtain the equilibrium quantities

$$x_2^{A*} = \frac{\alpha}{2+\gamma} + \frac{\theta_2}{2-\gamma} \qquad \text{and} \qquad x_2^{B*} = \frac{\alpha}{2+\gamma} - \frac{\theta_2}{2-\gamma}.$$

Quantity Setting in Stage $t = 1$

Firm A's expected profit considered in stage $t = 1$ is given by

$$\pi_1^A = x_1^A p_1^A + \delta E[\pi_2^A | I_1] = x_1^A \cdot (\alpha + \theta_1 - x_1^A - \gamma x_1^B) + \delta E[p_2^A x_2^A | I_1].$$

In period 1, firms anticipate the equilibrium quantities from period 2, so that $E[p_2^A(x_2^*)|I_1] = \frac{\alpha}{2+\gamma} + \frac{\theta_2}{2-\gamma} = x_2^{A*}$, and thus $E[\pi_2^A|I_1] = (x_2^{A*})^2$. We can additionally use the observations that $\theta_1 = E[\theta_1^*|I_1] = 0$ and

$$\partial\theta_2/\partial x_1^A = -\partial\theta_2/\partial x_1^B = \partial\eta_2/\partial x_1^A = (\gamma - 1)\tfrac{n_2}{2m_1}.$$

Using $\partial E[\pi_2^A|I_1]/\partial x_1^A = 2x_2^A \cdot \frac{1}{2-\gamma} \cdot \frac{\partial\theta_2}{\partial x_1^A}$, we obtain the FOC of firm A, given by

$$\frac{\partial \pi_1^A}{\partial x_1^A} = \alpha - 2x_1^A - \gamma x_1^B + \delta\left(\frac{2x_2^A}{2-\gamma} \cdot \frac{\partial\theta_2}{\partial x_1^A}\right) = 0.$$

This yields the best responses

$$x_1^A(x_1^B) = \frac{\alpha}{2} - \frac{\gamma x_1^B}{2} + \delta\left(\frac{2x_2^A}{2-\gamma} \cdot \frac{\partial\theta_2}{\partial x_1^A}\right), \qquad \text{and similarly} \qquad x_1^B(x_1^A) = \frac{\alpha}{2} - \frac{\gamma x_1^A}{2} + \delta\left(\frac{2x_2^B}{2-\gamma} \cdot \frac{\partial\theta_2}{\partial x_1^A}\right).$$

In equilibrium best responses intersect, and, using $E[\theta_2^*|I_1] = m_1 E[q] + n_2 E[\tilde{q}] = 0$, the equilibrium quantities are then given by

$$x_1^{A*} = x_1^{B*} = \frac{\alpha}{2+\gamma}\left(1 + \frac{\delta(\gamma-1)}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right).$$

Differentiation Investment in Stage $t = 0$

It holds that for $j \in \{A, B\}$

$$\frac{d(x_{NL}^{j*})^2}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j}\bigg|_{\gamma=1} = \frac{d(x_2^{j*})^2}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j}\bigg|_{\gamma=1} = \frac{2\delta\lambda\alpha^2}{27}.$$

Further helpful results for the calculation of the model with consumer learning are

$$\frac{dx_1^{j*}}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j} = \left[\frac{-\alpha}{(2+\gamma)^2} + \frac{\alpha\delta n_2}{m_1}\left\{\frac{(2+\gamma)^2(2-\gamma) - (\gamma-1)[2(2+\gamma)(2-\gamma) - (2+\gamma)^2]}{(2+\gamma)^4(2-\gamma)^2}\right\}\right] \cdot (-\lambda\gamma)$$

and

$$\frac{\partial[\alpha - (1+\gamma)x_1^{j*}]}{\partial k^j} = \lambda\gamma x_1^{j*} - (1+\gamma)\left[\frac{dx_1^{j*}}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j}\right].$$

Evaluating at $\gamma = 1$, using the sum rule in differentiation and the above results, we obtain

$$\frac{\partial\{x_1^{j*} \cdot [\alpha - (1+\gamma)x_1^{j*}]\}}{\partial k^j}\bigg|_{\gamma=1} = \frac{\lambda\alpha^2}{27}\left(1 - \frac{\delta n_2}{m_1}\right) + \frac{\lambda\alpha^2}{9} \cdot \left(1 - \frac{2}{3}\left\{1 - \frac{\delta n_2}{m_1}\right\}\right)$$
$$= \frac{\lambda\alpha^2}{9} \cdot \left(\frac{2}{3} + \frac{\delta n_2}{3m_1}\right).$$

Overall, the derivative of profit w.r.t. investment in differentiation, evaluated at $\gamma = 1$, is

$$\partial\pi_L^j/\partial k^j\big|_{\gamma=1} = \left(\frac{\partial\left[x_1^{j*}\{\alpha - (1+\gamma)x_1^{j*}\}\right]}{\partial k^j} + \delta \cdot \frac{d(x_2^{j*})^2}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j} - 1\right)\bigg|_{\gamma=1}$$
$$= \frac{\lambda\alpha^2}{9} \cdot \left(\frac{2}{3} + \frac{\delta n_2}{3m_1} + \frac{2\delta}{3}\right) - 1.$$

Appendix C Firm Behavior in the Bertrand Model

The calculations on the price setting in stages one and two of the Bertrand model can be done analogously to the quantity setting in the Cournot model (see Appendix B), and thus we will only calculate the optimizing behavior for differentiation investment in stage $t = 0$. In the following, the profit functions Π_k^j represent the same profits as in the Cournot model, only adapted to the Bertrand setting.

Differentiation Investment Without Consumer Learning (Benchmark)

No consumer learning implies $\theta_2 = 0$, such that the resulting optimal quantities and prices for firm A are

$$E[x_2^A(p_2^{A*})|I_1] = \frac{\alpha}{(2-\gamma)(1+\gamma)}$$
$$E[p_2^{A*}|I_1] = \frac{\alpha(1-\gamma)}{(2-\gamma)}.$$

The profit of firm A in a benchmark model with two periods without consumer learning is thus given by

$$\Pi_{NL}^j = (1+\delta) \cdot E[\Pi_2^A|I_1] - k^A = (1+\delta)E[x_2^A(p_2^{A*})|I_1] \cdot E[p_2^{A*}|I_1] - k^A.$$

The derivative of the objective function is then

$$\partial\Pi_{NL}^j/\partial k^j = (1+\delta) \cdot \frac{d(E[x_2^A(p_2^{A*})|I_1] \cdot E[p_2^{A*}|I_1])}{d\gamma} \cdot \frac{\partial\gamma}{\partial k^j} - 1$$
$$= (1+\delta) \cdot \frac{-\alpha^2(\gamma^3 - 3\gamma^2 + 4) + (3\gamma^2 - 6\gamma)(\alpha^2(1-\gamma))}{(\gamma^3 - 3\gamma^2 + 4)^2} \cdot (-\lambda\gamma) - 1$$

Firm A will invest in differentiation in equilibrium if

$$\partial \Pi^j_{NL}/\partial k^j\big|_{\gamma=1} > 0 \Leftrightarrow \lambda > \frac{2}{\alpha^2(1+\delta)} = \bar{\lambda}^B_{NL}.$$

Differentiation Investment With Consumer Learning

We can write

$$\begin{aligned} p_1^{A*} &= \frac{a}{2b-c} \cdot \left(1 - \frac{(b+c)b\delta}{4b^2-c^2} \cdot \frac{n_2}{m_1}\right) \\ &= \frac{\alpha(1-\gamma)}{2-\gamma} \cdot \left(1 - \frac{(1+\gamma)\delta}{4-\gamma^2} \cdot \frac{n_2}{m_1}\right) \\ &= \frac{\alpha(1-\gamma)}{2-\gamma} - \frac{(1-\gamma^2)\delta\alpha}{(4-\gamma^2)(2-\gamma)} \cdot \frac{n_2}{m_1}. \end{aligned}$$

The profit of firm A in the model with consumer learning is given by

$$\begin{aligned} \Pi^j_L &= p_1^{A*} \cdot \left[a + (c-b)p_1^{A*}\right] + \delta \cdot E[x_2^A(p_2^{A*})|I_1] \cdot E[p_2^{A*}|I_1] - k^A \\ &= p_1^{A*} \cdot \left[\frac{\alpha}{1+\gamma} + \frac{\gamma-1}{1-\gamma^2} \cdot p_1^{A*}\right] + \delta \cdot E[x_2^A(p_2^{A*})|I_1] \cdot E[p_2^{A*}|I_1] - k^A. \end{aligned}$$

Further helpful results are

$$\frac{\partial p_1^{A*}}{\partial \gamma} = \frac{-\alpha(2-\gamma) + \alpha(1-\gamma)}{(2-\gamma)^2} - \frac{(-2\gamma\delta\alpha)(2-\gamma)(4-\gamma^2) - (3\gamma^2-4\gamma-4)\alpha\delta(1-\gamma^2)}{(2-\gamma)^2(4-\gamma^2)^2}$$

$$\begin{aligned} \frac{\partial\left[\frac{\alpha}{1+\gamma} + \frac{\gamma-1}{1-\gamma^2} \cdot p_1^{A*}\right]}{\partial k^A} &= -\alpha + \frac{(1-\gamma^2) + 2\gamma(\gamma-1)}{(1-\gamma^2)^2} \cdot p_1^{A*} + \frac{\gamma-1}{1-\gamma^2} \cdot \frac{\partial p_1^{A*}}{\partial \gamma} \\ \left[\frac{\alpha}{1+\gamma} + \frac{\gamma-1}{1-\gamma^2} \cdot p_1^{A*}\right]\Bigg|_{\gamma=1} &= \alpha/2 \\ p_1^{A*}\Big|_{\gamma=1} &= 0 \\ \frac{\partial p_1^{A*}}{\partial \gamma}\Bigg|_{\gamma=1} &= -\alpha + \frac{2\alpha\delta n_2}{3m_1} \end{aligned}$$

Overall, the derivative of profit w.r.t. investment in differentiation, evaluated at $\gamma = 1$, is

$$\begin{aligned} \partial \Pi^j_L/\partial k^j\big|_{\gamma=1} &= \alpha\lambda\left(1 - \frac{2\delta n_2}{3m_1}\right) \cdot \frac{\alpha}{2} + 0 \cdot \frac{\partial\left[\frac{\alpha}{1+\gamma} + \frac{\gamma-1}{1-\gamma^2} \cdot p_1^{A*}\right]}{\partial k^A}(-\lambda) + \delta\lambda\frac{\alpha^2}{2} - 1 \\ &= \left[\frac{\alpha^2}{2}(1+\delta) - \frac{\alpha^2\delta n_2}{3m_1}\right]\lambda - 1. \end{aligned}$$

Firm j will invest in differentiation in equilibrium if

$$\begin{aligned} & \partial\Pi_L^j/\partial k^j\big|_{\gamma=1} > 0 \\ \Leftrightarrow \qquad & \lambda > \frac{2}{\alpha^2\left[(1+\delta) - 2\delta n_2/(3m_1)\right]} := \bar{\lambda}_L^B. \end{aligned}$$

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